Energy-Based Metrics for General Aviation Flight Data Record Analysis

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Energy management and energy state awareness are important concepts in aircraft safety analysis. Many loss-of-control accidents can be attributed to poor energy management. Energy-based metrics provide a measurable quantity of the energy state of the aircraft and can be viewed as an objective currency to evaluate various safety-critical conditions. In this work, we have surveyed key energy-based metrics from various domains and identified the challenges of implementing these metrics for General Aviation operations. Modifications to existing metrics and definition of some new energy metrics are proposed. A methodology is developed that can be used to evaluate and visualize the energy metrics. These energy metrics can then be used to understand and enhance General Aviation aircraft safety using retrospective flight data analysis.

Nomenclature

D	Drag
E	Specific total energy
g	Acceleration due to gravity
h	Altitude
m	Mass, kg
T	Thrust
V	Velocity
W	Weight of the aircraft
δE	Specific energy error
$\dot{\delta E}$	Energy error rate
E_{tol}	Tolerance in energy error
δE_n	Normalized energy error
$\dot{E}$	Total energy rate
P_s	Specific excess power
γ	Flight path angle
γ_p	Potential flight path angle
γ_E	Total energy angle
$\eta_{\dot{E}}$	Energy rate efficiency
$\hat{E}$	Energy rate demand
a_m	Acceleration during Turn
e	Specific energy to turn
ω	Angular velocity during turn

I. Introduction

There is a great impetus to improve safety across all flight regimes in General Aviation (GA) operations. Energy state awareness and energy management are critical concepts in the characterization, detection, and prevention of safety-critical conditions. Poor energy management and loss of energy state awareness have been shown to be top contributors to Loss of Control (LoC)[1] and controlled flight into terrain (CFIT), recognized by the FAA as the leading causes of fatal accidents in general aviation, and by others[2] as leading causes among all aircraft types, operations, and phases of flight. Paradoxically, energy state awareness and management have been addressed almost exclusively in commercial aviation where the concepts are intrinsic in operational safety and have been the subject of much research. Nevertheless, the General Aviation Joint Steering Committee (GAJSC) has identified various safety enhancements for new and current GA aircraft intended to improve state awareness such as angle of attack systems, stall margin indicators, and stabilized approach indicators.[3]

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We assert that energy-based metrics, namely those that characterize the energy state and safety boundary conditions of the aircraft, hold significant potential for improving GA operational safety because they explicitly address poor energy management and state awareness as the top contributing factors underlying LOC and CFIT events.

We further assert that energy-based metrics have no loss generality in the characterization of aircraft states and safety boundaries across the GA fleet, and are therefore preferred over flight parameters such as angle of attack, stall margin, or rate of descent. Whereas safety boundaries expressed with such flight parameters change from one aircraft to another and states may not be directly comparable, energy metrics provide a common and objective language that is broadly applicable across the spectrum of a very heterogeneous GA fleet.

Lastly, we posit that Flight Data Monitoring (FDM) is extremely well suited as a means to introduce energy management and state awareness to the GA community, and to adopt energy-based metrics as a staple in the state of the practice of operational safety assessments. FDM are voluntary safety programs that aim to improve operational safety with a continuous cycle involving data collection from on-board recorders, retrospective analysis of flight data records, identification of operational safety exceedances, design and implementation of corrective measures, and monitoring to assess their effectiveness. In current FDM practice safety events are defined *a priori* as the concurrent exceedance of one or more flight parameters over corresponding threshold values. Implementation of FDM is widespread in commercial aviation, and although it is sparse among GA operators, recent and current efforts seek its introduction and broad adoption in that sector.

In consideration of the above we articulate the needs motivating the work here reported, as follows. First, suitable energy-based metrics are requisite for retrospective monitoring of energy management in FDM. They must provide an objective and meaningful quantification of the aircraft's energy state, safety boundaries, and departure from safe or nominal conditions, that is consistent with flight parameters recorded. Second, implementation and evaluation of energy metrics in FDM data analysis using real flight data records is necessary to demonstrate its feasibility and outline practical considerations.

This paper presents key outcomes and contributions to the state of art resulting from our efforts to address the above needs. In Section II we present an exhaustive survey and review of the body of work on aircraft energy state characterization. We describe a wealth of energy-based metrics and propose an exhaustive classification scheme based on their definition, purpose, and underlying energy concept. We then outline intrinsic challenges in the application of these energy metrics to FDM analysis for GA operations in Section III. We demonstrate the implementation of energy metrics for FDM analysis on a large set of flight data records in Section IV. There we present our approach to define a nominal trajectory which forms the basis of evaluation for a subset of energy metrics. We also address the use of specific flight parameters contained in data records and of validated GA aircraft performance models developed in prior work.[4,5] New energy metrics developed in this work, either as all-new metrics or variants of existing metrics, are introduced and discussed. We further demonstrate the visualization of all energy metrics as data time-series for large sets of data records, and illustrate its use for the visual inspection of individual records where anomalous energy states may be readily identified. A summary of observations, findings, and an outline for future work is presented in Section V.

II. Review of Energy Metrics and their Applications

A. Classification and Organization of the Literature

The literature on energy management outlines two fundamental objectives at the highest level - improving safety and efficiency.[6] Within the literature targeting safety improvements, there are three major themes used to outline the scope of each effort: the specific purpose, the intended aircraft category, and the flight regime. We organize the literature surveyed according to these themes as follows:

Classification based on Aircraft Category

1. Commercial aircraft[1,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20]
2. Fighter aircraft[21,22]
3. General Aviation[23]

4. Simulator[24]
5. Unmanned Aerial Vehicles[25,26,27]

Classification based on Flight Regimes

1. Approach, descent, and landing[8,12,13,14,15,19,20,23,24,28]
2. Cruise[15]
3. General[1,6,7,9,10,11,16,17,18,21,25,26,27,29]
4. Turning and maneuvering[22,30]

Classification based on Purpose

1. Development of cockpit flight displays for enhancing energy awareness of crew[8,9,23,21,24,29]
2. Pilot training for better energy management[6]
3. Development of control system based on energy management[7,9,11,16]
4. Trajectory optimization algorithms[25]
5. Energy management during descent or defining descent trajectories such as minimum fuel consumption, low noise, emissions and various other types of trajectories[12,13,14,15,19,20,24,28]
6. Analysis of accidents or incidents involving poor energy state management.[1,17,18]

From the above groupings, some immediate trends are apparent. Most notably, it is observed that a majority of the work done on energy metrics has been performed with commercial aviation in mind. Additionally, while most work is not specific to a given flight regime (denoted by 'General' above), phase-specific metrics are most often developed for descent, approach, and landing, consistent with the significant number of accidents and incidents that are known to occur in these phases. With regard to the overall purpose of each work, a broad spread of applications is observed, suggesting a variety of approaches to the problem of energy-based safety assessments.

B. Energy-based Metrics in Literature

Figure 1 illustrates our organization and categorization of energy-based metrics in literature. It also lists the various cockpit displays used for energy management, and which metrics they utilize. As noted in Figure 1, energy-based metrics in literature can be divided into 2 main categories:

1. Metrics related to Total Energy
2. Metrics related to Total Energy Rate

There are a few other energy based-metrics in literature that do not fall clearly in either these two categories, and that have grouped under the *Other Metrics* category. Most work related to energy metrics or energy management use simple energy metrics that can be obtained from the kinematics of the aircraft (altitude, velocity and their rates). Some of the metrics might require the definition of a reference profile to compare the energy state against (e.g.: Energy Rate Demand, Energy Rate Efficiency).

The metrics listed in Figure 1 under Total Energy Metrics and Total Energy Rate Metrics are calculated at each point of time during the flight, and can be displayed on a cockpit monitor. Therefore, they characterize an *instantaneous* energy state of the aircraft. Other metrics provide measures of energy states, or compliance to nominal states, *aggregated* over a time period. For instance, the mean of the absolute value of energy/energy error[8] measures how well a pilot followed a certain profile over a particular phase of flight. For the application considered in this paper (which is retrospective flight data analysis), both instantaneous and aggregated metrics need to be considered. The remaining part of this section focuses on the energy metrics used in literature and their definitions.

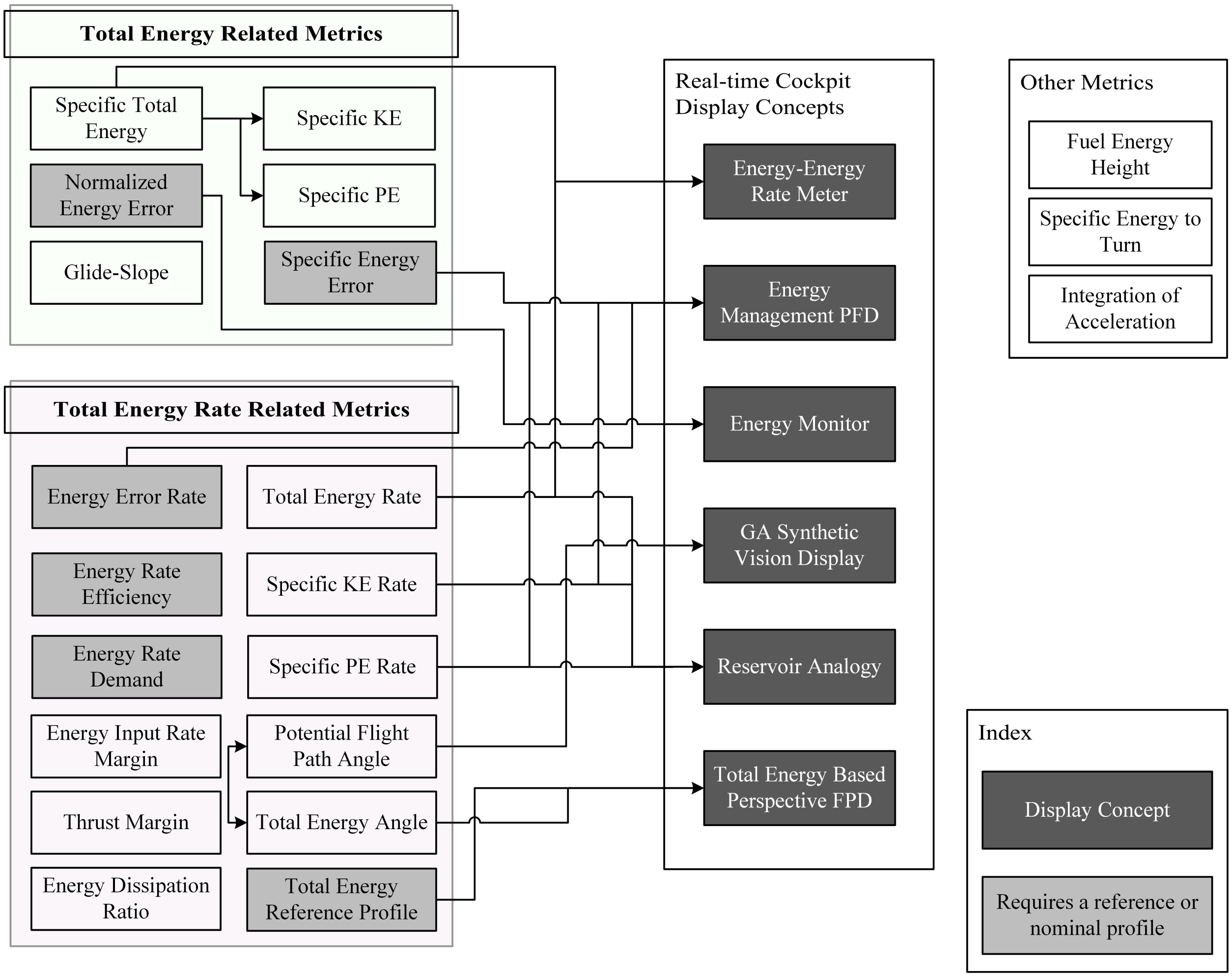


Figure 1. Summary of energy based metrics and display concepts in literature

Total Energy Metrics

For defining total energy-based metrics, the total mechanical energy (kinetic + potential) of the aircraft is either used directly or compared against a reference value to obtain the metric.

1. *Specific Total Energy*

This is one of the most widely used metrics in literature.[6,21,11,17,18] Specific total energy is usually the metric representing the energy state of the aircraft as defined in many papers. It is also known as Energy Height. Specific total energy is given by:

$$E = h + \frac{V^2}{2g} \tag{1}$$

This metric has been used in a display concept called Energy/Energy Rate meter by Zagalsky.[21] Specific Total Energy has also been extensively used by Rutowski[31] in a graphical method to determine the optimum flight profile for an aircraft to reach a certain speed and altitude. Boyd[30] used Energy-Maneuverability Theory to generate "sky-maps" for candidate aircraft. One of the important parameters used on these sky-maps was the energy height. This metric can be completely obtained from the state of the aircraft and does not require a reference flight profile. Specific total energy by itself is not a very useful metric as it can only give an indication of the total energy of the aircraft and not how it is distributed.

2. ***Specific Potential Energy***

Specific Potential Energy is defined as the potential energy per unit weight of the aircraft. It is given by:

$$PE = \frac{mgh}{W} = h \tag{2}$$

3. ***Specific Kinetic Energy***

Specific Kinetic Energy is defined as the kinetic energy per unit weight of the aircraft. It is given by:

$$KE = \frac{\frac{1}{2}mV^2}{W} = \frac{V^2}{2g} \tag{3}$$

Specific Potential Energy and Specific Kinetic Energy are usually a part of any formulation that involves the Specific Total Energy since these are the two components that make up the Specific Total Energy. In addition, the Specific Potential Energy (in the form of altitude) and the Specific Kinetic Energy (in the form of airspeed) are part of any cockpit display for most aircraft. These two metrics give an indication of the aircraft's mechanical energy distribution and can be important indicators for operational/safety related events.

4. ***Specific Energy Error***

Specific Energy error is defined as the difference in specific total energy in actual flight to that of a reference profile. It is given by:

$$\delta E = E_{act} - E_{ref} = \underbrace{h_{act} - h_{ref}}_{\text{PE Error}(\delta E_{pot})} + \underbrace{\frac{V_{act}^2 - V_{ref}^2}{2g}}_{\text{KE Error}(\delta E_{kin})} \tag{4}$$

This metric is very widely used for commercial aircraft, especially in descent and landing where a reference flight profile is available. Williams et al.[13] have used specific energy error to compare various descent trajectories. Jong et al.[12] have used specific energy error as a metric in developing a planning and guidance concept for optimizing aircraft trajectories during descent. Amelink et al.[29] have used total energy deviation and kinetic energy deviation in the formulation of their *Total Energy Based Perspective Flight Path Display.* Lambregts[9] has used specific energy error in his *Energy Management Primary Flight Display* concept. Other works which include energy error as a metric in their formulation include the works of Williams.[14,15]

This metric is very useful for defining deviations from a reference trajectory when such a trajectory is available. Reference trajectories for both potential and kinetic energy must be available. The specific energy error can be further divided into the specific potential energy error (δE_{pot}) and specific kinetic energy error (δE_{kin}).

5. ***Normalized Energy Error***

Normalized Energy Error is a metric very similar to Specific Energy Error. It is the specific energy error normalized by a tolerance on the specific energy error. This metric has been used by Gandhi et al.[8] in the development of their "Energy Monitor" display concept. Normalized Energy Error is given by equation 5:

$$\delta E_n = \frac{E - E_{ref}}{E_{tol}} \tag{5}$$

The tolerance in energy error (E_{tol}) is updated dynamically as the aircraft tries to follow a reference approach profile. The authors have developed a crew alerting system which provides various cues when the normalized error exceeds a certain threshold.

Total Energy Rate Metrics

Similar to Total Energy Metrics described earlier, these metrics use the rates of change of total, potential, and kinetic energy. These rates may be directly used as a metric or they may be compared to reference/threshold values.

6. ***Specific Total Energy Rate***

Specific Total Energy Rate which is also rate of change of energy height is defined as the rate of change of Specific Total Energy. It is given by:

$$\dot{E} = \frac{dE}{dt} = \dot{h} + \frac{V \times \dot{V}}{g} \tag{6}$$

It is also called Specific Excess Power[6,30] and can be alternatively represented as:

$$\dot{E} = \frac{(T - D)V}{W} = P_s \tag{7}$$

Equation 6 is the energy rate from the aircraft state point of view (altitude and velocity), whereas Eq. 7 is from the aircraft systems point of view (Propulsion/Thrust and Aerodynamics/Drag). Zagalsky[21] has used Specific Energy Rate in their Energy/Energy Rate meter display concept. Lambregts[7] has used Specific Energy Rate in his formulation of the Total Energy Control System concept.

7. ***Specific Potential Energy Rate***

Specific Potential Energy Rate (SPER) is given by:

$$SPER = \dot{h} = V \times \sin\gamma \tag{8}$$

8. ***Specific Kinetic Energy Rate***

Specific Kinetic Energy Rate (SKER) is given by:

$$SKER = \frac{d(\frac{V^2}{2g})}{dt} = \frac{V \times \dot{V}}{g} \tag{9}$$

Both Specific Potential Energy Rate and Specific Kinetic Energy Rate are typically used in conjunction with the Specific Total Energy Rate. The Reservoir Analogy[29,6] uses the Specific Potential and Kinetic Energy rates along with the Specific Total Energy rate to show how the energy entering the system is being distributed. Lambregts[9] uses these metrics in his new ecological primary flight display concept.

9. ***Potential Flight Path Angle/Total Energy Angle***

The Potential Flight Path Angle (hereafter referred to as PFPA) is a measure of the attainable flight path angle at the current throttle setting. It is the flight path angle that the aircraft can attain when there is no acceleration along the flight path. PFPA is given by:

$$\gamma_p = \gamma + \frac{\dot{V}}{g} \tag{10}$$

It is a measure of the dimensionless total specific energy rate and can be related to the total specific energy rate as:

$$\gamma_p = \frac{\dot{E}}{V} \tag{11}$$

The upper limit associated with PFPA is the maximum potential flight path angle ($\gamma_{p,max}$) which represents the flight path angle that can be achieved at the theoretical maximum thrust while maintaining the current speed and current aircraft configuration. Similarly, the minimum PFPA ($\gamma_{p,min}$) is the flight path angle that can be achieved with idle thrust setting while maintaining current speed and aircraft configuration.

PFPA has been widely used in literature as an energy metric for cockpit displays. Adami et al.[23] have used PFPA in their "General Aviation Synthetic Vision Display" concept. Tadema et al.[26] have used PFPA in their display concept "Traffic Terrain and Energy Awareness Display for UAV". Lambregts et al.[27] have used PFPA and PFPA-max in their work on investigating use of full lateral and vertical control authority for UAV conflict resolution. This work has explored the possibility of the flight path angle going beyond the maximum PFPA ($\gamma_{p,max}$) in certain maneuvers. Lambregts[7] and Kurdjukov et al.[11] have used PFPA in the formulation of the Total Energy Control System. PFPA has been used under the name *Total Energy Angle* by van den Hoven et al.[24] and Amelink et al.[29] in the description of the *Total Energy Based Perspective Flight Path Display.*

10. ***Energy Error Rate***

Energy Error Rate is a metric that has been used by Williams.[13] This metric provides a measure of whether the Energy Error with respect to a reference profile is increasing or decreasing. This metric is given by:

$$(\dot{\delta E}) = \frac{\Delta(\delta E)}{\Delta t} \tag{12}$$

The authors have highlighted that the energy error rate when presented as a trend arrow on cockpit displays aids pilots in flying the reference profile while staying within the limits of energy error which are dynamically updated.

11. ***Energy Rate Efficiency***

Energy Rate Efficiency is a measure of how closely an aircraft is following the commanded energy profile of an approach trajectory. It is given by:

$$\eta_{\dot{E}} = \frac{\dot{E}_{cmd}}{\dot{E}_{tot}} = \frac{V_c W(\gamma_c + \frac{\dot{V}_c}{g})}{V_a(T - D)} \tag{13}$$

The Energy Rate Efficiency has been used by van den Hoven et al.[24] to analyze approach trajectories. When this metric is equal to unity, the aircraft is following the commanded trajectory exactly. A value higher than unity indicates a deficit of total energy and value lower than 1 indicates excess total energy than what is required by the approach profile. This metric does not yield meaningful results when there is no ascent/descent or acceleration (such as steady level flight).

12. ***Energy Rate Demand***

Energy Rate Demand is the maximum energy dissipation that the aircraft can attain at the current speed and configuration. This is used when the aircraft is descending. It is given by:

$$\hat{E} = \frac{W(\gamma_c + \frac{\dot{V}_c}{g})}{T_{idle} - D} \qquad \text{Descending flight} \tag{14a}$$

$$\hat{E} = \frac{W(\gamma_c + \frac{\dot{V}_c}{g})}{T_{max} - D} \qquad \text{Ascending flight} \tag{14b}$$

When the Energy Rate Demand goes above unity, it indicates that the aircraft, in its current configuration, cannot fly the commanded trajectory. Energy Rate Demand has been used in literature by van den Hoven et al.[24] and Amelink et al.[29] It has also been used as a constraint by Vormer et al.[19] to ascertain which profile can and cannot be flown by an aircraft during flexible approach trajectory optimization.

13. ***Total Energy Reference Profile***

The Total Energy Reference Profile (TERP) is defined as the energy profile followed by the aircraft when the total energy error (δE - defined earlier in Eq. 4) from a commanded profile is zero. When the aircraft is flying along the TERP, the Potential Flight Path Angle is equal to the Potential Flight Path Angle of the reference trajectory. ($\gamma_p = \gamma_{p,c}$). Van den Hoven et al.[24] and Amelink et al.[29] have used Total Energy Reference Profile in the development of their "Total Energy Based Perspective Flight-Path Display".

Other Metrics

Apart from the metrics described above, there are other metrics covered in literature that are related to energy state. They have been elaborated here:

14. ***Fuel Energy Height***

Pennycuick[10] has defined a metric called Fuel Energy Height. This metric is the total specific mechanical energy plus the chemical energy contained in the fuel which is converted into an equivalent height assuming some efficiency of converting the the chemical energy into potential energy. Thus the fuel energy height gives an absolute upper bound on the total energy available to an aircraft at any point in time.

15. ***Integration of Acceleration history***

Anderson et al.[25] have used the integration of acceleration during turns as one of the objectives in a multi-objective optimization of UAV trajectory. This is given by:

$$\int_0^t a_m^2 dt \tag{15}$$

where a_m is the acceleration during the turn. The idea behind this metric is that it is a surrogate for the energy expended during all the turns.

16. ***Specific Energy to Turn (SET)***

Yajnik[22] has defined a metric to evaluate efficiency of turning flight. It is the energy required to overcome air resistance per unit mass per turn. This metric is given by equation 16

$$e = (DV)\frac{g}{W}\frac{2\pi}{\omega} \tag{16}$$

Turning with the minimum specific energy to turn is desired and this metric would prove useful in these situations. This metric has been utilized by the author in the form of SET-turn rate graph for various trade studies on important conceptual design parameters such as wing loading, zero-lift drag coefficient, and others.

17. ***Energy-Maneuverability and Sky-Maps***

Rutowski[31] had used contours of total energy to calculate minimum time to climb using a graphical method. Boyd[30] extended this to the Energy-Maneuverability theory to rank relative performance of combat aircraft. Recently, Takahashi[32] has used the concept of "sky-maps" for visualizing various aircraft performance parameters and capabilities, including some of the energy metrics discussed earlier.

III. Challenges and Opportunities for General Aviation Application

In order to enhance GA aircraft safety with the surveyed energy-based metrics, it is important to be aware of the challenges of implementing the energy-based metrics into GA aircraft safety enhancement efforts. Several challenges were identified and have been enumerated below:

1. To define some of the energy metrics during approach and landing, a nominal or reference profile of altitude and velocity is desired. The energy metrics represent how well the aircraft is adhering to these reference profiles and whether the aircraft can execute the trajectory. The reference profiles are usually well defined for commercial aircraft operations but are not so clearly defined in the GA aircraft category. Therefore, it is important to identify a way of defining such nominal profiles to aid in the use of energy metrics. This problem has been addressed by the authors in a parallel effort.[33]

2. In addition to the metrics defined earlier, defining the limits of aircraft operation is also important. V-n diagram, excess power plot, energy height diagram, and turn rate envelope represent the operating envelope of an aircraft. These envelopes indicate whether the aircraft is operating away from its limits. Aircraft operations within these envelopes are considered safe and permissible. However, there is ambiguity regarding the determination of the extent to which the aircraft state is safe or allowable. Therefore, energy metrics to measure the offset from the limits are required. It is important to distinguish what limit is more critical to the cause of accidents or incidents.

3. Recoverability is an important concept to measure or determine how safe the aircraft is in its envelope. For example, an aircraft may cross stall limit if the aircraft is in high total energy state, but it should not cross the line in low energy state. Thus, metrics that not only define aircraft-specific operating envelopes, but also combine different envelopes need to be established in order to quantitatively determine the safety of an aircraft.

4. Even with the availability of a nominal profile, the implementation of energy metrics should be meaningful and insightful. Identifying limits/thresholds on the values of energy metrics can help enhance/augment efforts to increase safety.

5. Energy metrics can also be used in a Flight Data Monitoring setting to identify unsafe/anomalous flight data records. This can be done in various ways and compared to traditional methods such as exceedance detection.

Although this work focuses on overcoming some portions of the challenges listed above and does not address all of them in this paper, it is worthwhile to recognize the challenges in order to facilitate future areas of research in this domain.

IV. Implementation of Energy Metrics

The methodology outlined in Figure 2 is utilized to implement energy metrics identified earlier. It is important to note that, unlike traditional approaches of using energy metrics, this work focuses on using energy metrics in a retrospective flight data analysis setting. The methodology described here can be used to evaluate the energy metrics for flight records and visualize them. These defined metrics can then be used all at once (or a subset) to understand and enhance GA aircraft safety.

The important components of the methodology include defining nominal profiles for velocity and altitude, the performance models used, the metric evaluation for each flight record, and the visualization and safety analysis possible using this method. Each individual flight record is a part of a data set of over six hundred records collected from training flights using a Cessna 172S aircraft equipped with Garmin G1000 for flight data collection. The methodology is applied to the approach and landing phase initially with the understanding that it can be similarly extended to other phases of flight. Since many GA accidents occur during approach and landing,[18] these phases were chosen to test the approach.. The following subsections elaborate on each of these important components.

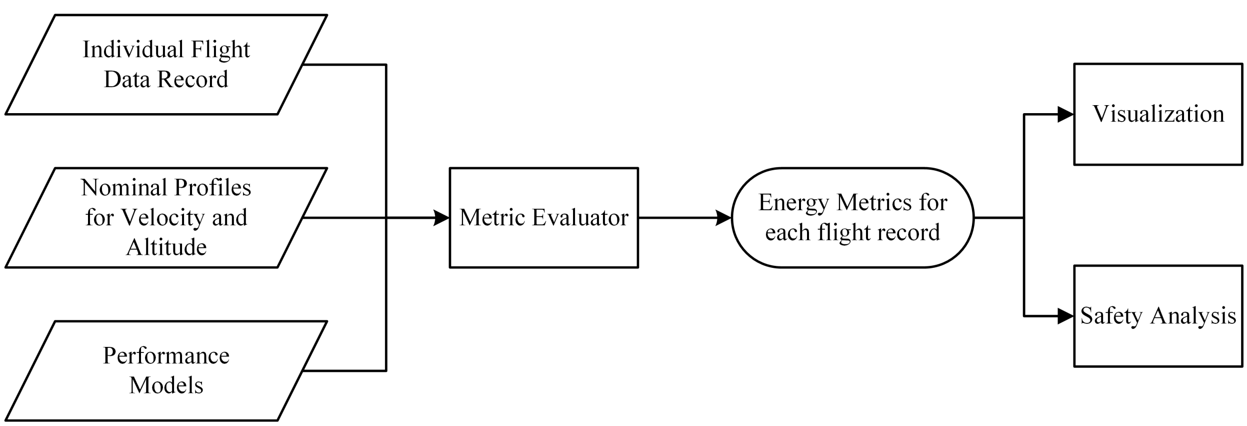
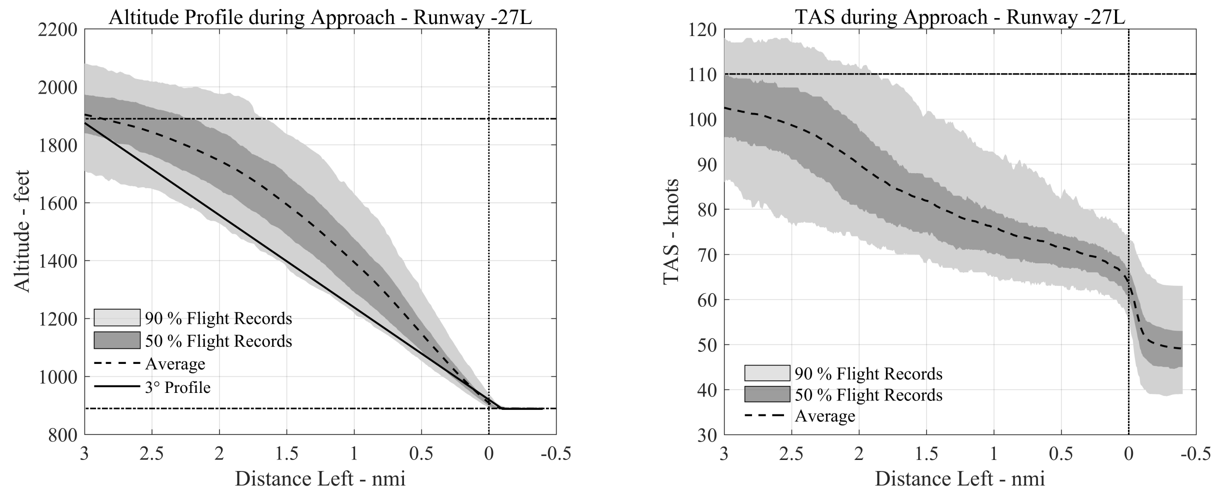


Figure 2. Outline of the methodology and steps involved in the process

A. Defining a Nominal Trajectory

In other work by the authors in Puranik et al.,[33] various aspects of defining a nominal approach and landing trajectory for GA applications are explored. As noted previously, defining a nominal profile for altitude and velocity is important for defining some energy metrics identified from literature. This profile is defined using a statistical approach by averaging the altitude and velocity over a large data set of flights. In order to ensure that this averaging is done such that different records can be compared against each other, the flight record is sampled based on distance remaining to touchdown rather than time. This allows discretizing the approach and landing phase into extremely small segments and averaging the values of altitude and velocity at each point. The results of this analysis are shown in Figure 3.



(a) Average/Nominal altitude profile

(b) Average/Nominal velocity profile

Figure 3. Finding a nominal trajectory using flight records

As is evident from the altitude profile and spread of altitude for flights, the typical 3° profile is typically not strictly followed by most flights. Most of the flight records intercept this profile at approximately 1000 feet above ground level (horizontal dashed line on top). For the velocity profile, it is observed that the average velocity drops considerably during approach and landing and shows variability at the runway threshold. Also, there are no noticeable steps in the velocity profile suggesting against a stepped approach.

Due to the observations noted in this section, it was important to obtain the nominal profile as the statistical average over a large set of flights. Further insights and details on the implementation can be found in Puranik et al.[33] For the purpose of implementing energy metrics, the average profiles are used.

B. Description of Performance Models

In order to evaluate some of the energy metrics listed earlier in Section II we need additional data beyond what is captured by the typical on-board flight data recorder on GA aircraft. This includes the thrust and drag of the aircraft at any given point of time. Previous work by the authors[4,5] included development and validation of performance models for GA aircraft using publicly available data. These models are able to predict the lift, drag, thrust, and the theoretical maximum thrust of the aircraft at any given point in time. In addition, lift and drag of unclean configurations can also be predicted with these models. A brief description of the models is given here and readers are encouraged to refer Min et al.[4] for the aerodynamics model and Harrison et al.[5] for the propulsion model. It should be noted that in this methodology, other equivalent performance models could have been utilized which are able to provide validated predictions for the quantities of interest.

1. Aerodynamics

Since an accurate aerodynamic model of GA aircraft is crucial to the understanding of aircraft performance, several aerodynamic modeling and calibration methods for fixed wing aircraft were examined and compared in the previous study to generate an appropriate aerodynamic model for a specific GA aircraft. Lift-curve modeling consists of three stages and each stage has five parameters to define the curve. Drag polar can be defined with two parameters for simplified parabolic drag model. Thorough analysis on various modeling methods output an optimum set of parameters which minimizes modeling errors compared to any publicly available reference data. Based on the work in this published aerodynamic model, the aerodynamic model used in this paper was refined using hundreds flight data records by minimizing errors between modeling results and actual data-driven values. The inputs to the final model are the raw flight data and the outputs are the dimensional and non-dimensional lift and drag.

2. Propulsion

Alongside the aerodynamic forces of flight, an estimation of the propulsive characteristics creates a more complete picture of GA performance. The implemented propulsion model generates estimate of thrust produced by a fixed-pitch propeller driven by an internal combustion piston engine. First, a Otto-cycle based simulation of the engine is performed to generate the output power of the engine. Then this power is used by an empirical-based propeller performance model to estimate propeller thrust. Using these two validated models, estimations of thrust require the input of only five parameters: altitude, operating temperature, airspeed, engine RPM, and engine fuel-air ratio.

C. Evaluation of Energy Metrics

Using the nominal profiles defined and the performance models it is possible to evaluate all the relevant metrics from the literature survey for the approach and landing phase. Some metrics identified in literature tend to be numerically ill-behaved in the approach and landing phase. These need to be redefined in such a way that they are useful. The following subsection elaborates some of these modified metrics in addition to some newly defined metrics used in this approach.

Additional and Modified Energy Metrics

1. Inverse of Energy Rate Efficiency

Earlier, Energy Rate Efficiency was defined by Eq. 13 as the ratio between the specific energy rate of the commanded profile to the actual profile. However, during approach and landing (or take-off) operations in GA, at many points in time the actual total energy rate can be zero. This causes the energy rate efficiency to have sharp peaks and even be undefined at some places (division by zero).

On the other hand, as observed in Figure 4 the specific total energy rate of the nominal reference profile is never zero during approach and landing (it is always negative). Therefore, defining the energy rate efficiency as the inverse of what was defined in Eq. 13 is more meaningful and intuitive. This inverse energy rate efficiency is defined as:

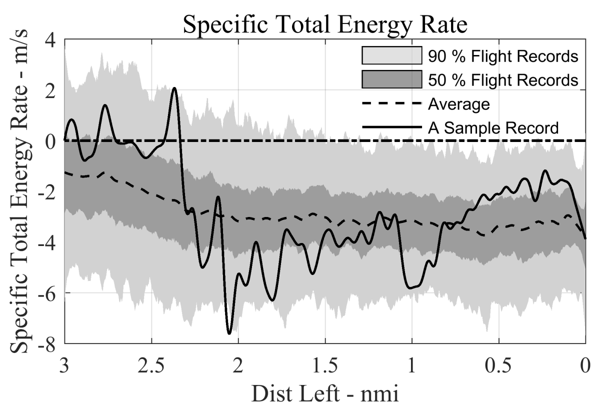


Figure 4. Specific Total Energy Rate of the nominal profile and a sample flight record

$$\eta_{\dot{E},inv} = \frac{\dot{E}_{tot}}{\dot{E}_{cmd}} = \frac{V_a(T-D)}{V_c W(\gamma_c + \frac{\dot{V_c}}{g})} \quad (17)$$

A value of this metric higher than unity would indicate a specific energy rate higher than the reference (and possibly point to unsafe states). On the other hand a negative value would indicate that the aircraft is gaining energy where the reference profile is losing (because it is applied to approach and landing phase) - again, possibly indicating a red flag in terms of safety. A value between 0 to 1 is preferred.

2. ***Modified Energy Error Rate***

The energy error rate is defined by Eq.12. This rate represents whether the total energy error with respect to the nominal profile is increasing or decreasing. However, just the error rate does not represent the entire picture. A positive energy error rate would be preferred if the energy error itself is negative and vice versa. Therefore, a modified energy error rate is proposed to be used which also takes into consideration the sign of the energy error. This is given by:

$$(\dot{\delta E})_m = sign(\delta E) \times \frac{\Delta(\delta E)}{\Delta t} \tag{18}$$

A negative value of the modified energy error rate is always preferred as this will mean that the error is being driven towards zero. While it is understood that the modified energy error rate will not always be zero, it is nevertheless desirable to have this metric within reasonable bounds. Another characteristic related to this metric is that it should not be positive for extended periods of time which would indicate that the current state trajectory is deviating away from the reference energy profile.

3. ***Thrust Margin***

The thrust margin metric is defined as the ratio of the current thrust to the theoretical maximum thrust possible at that flight condition. It is given by:

$$TM = 1 - \frac{T}{T_{max}} \tag{19}$$

The thrust margin is an indirect indicator of the amount of energy that can enter the system. Operating at higher margin would be preferable as it would mean that the aircraft can escape possible low-energy scenarios by the aggressive addition of energy.

4. ***Glide Slope***

The specific potential energy rate describes the change in potential energy with respect to time. It is also useful to find the rate of change of potential energy with respect to ground track distance covered (instantaneous glide slope). The glide slope metric is defined as the ratio of the the altitude change per unit distance covered along the ground track. It is given by:

$$GS = \frac{dh}{dx} \tag{20}$$

Rather than using glide slope, the error in glide slope (with respect to a 3° profile) can also be used.

5. ***Energy Rate Margin***

Energy Rate Margin is defined as the ratio of the actual specific energy rate to the specific energy rate using the theoretical maximum thrust value for the same configuration (same drag). It is given by:

$$\bar{E} = \frac{W(\gamma_a + \frac{\dot{V}_a}{g})}{T_{max} - D} \tag{21}$$

During approach and landing, the actual specific energy rate is expected to be negative whereas the maximum specific energy rate will be positive. Therefore, a small negative value (greater than -1) would indicate that the specific energy rate is negative but can be made positive at the current configuration by increasing the thrust. A value less than -1 would indicate that the specific energy rate is negative and the aircraft does not have sufficient thrust to make this positive. A value greater than zero would indicate that the aircraft has a positive specific energy rate instead of negative. The theoretical upper limit on this metric is +1.

Table 1. **Summary of implemented energy metrics, formulas, and data required for computation**

		Can be estimated using		
Metric	**Formula**	**Flight Data**	**Flight Data plus Performance Models**	**Requires Reference Profile**
Specific Total Energy	$h + \frac{V^2}{2g}$	Yes	Yes	No
Specific Potential Energy	h	Yes	Yes	No
Specific Kinetic Energy	$\frac{V^2}{2g}$	Yes	Yes	No
Specific Total Energy Error	$h_{act} - h_{ref} + \frac{V_{act}^2 - V_{ref}^2}{2g}$	Yes	Yes	Yes
Specific Potential Energy Error	$h_{act} - h_{ref}$	Yes	Yes	Yes
Specific Kinetic Energy Error	$\frac{V_{act}^2 - V_{ref}^2}{2g}$	Yes	Yes	Yes
Normalized Energy Error	$\frac{E - E_{ref}}{E_{tol}}$	Yes	Yes	Yes
Specific Total Energy Rate	$\dot{h} + \frac{V.\dot{V}}{g} = \frac{(T-D)V}{W}$	Yes	Yes	No
Specific Potential Energy Rate	$\dot{h} = V \sin\gamma$	Yes	Yes	No
Specific Kinetic Energy Rate	$\frac{V.\dot{V}}{g}$	Yes	Yes	No
Potential Flight Path Angle	$\gamma + \frac{\dot{V}}{g}$	Yes	Yes	No
Glide slope	$\frac{dh}{dx}$	Yes	Yes	No
Modified Specific Energy Error Rate	$\mathrm{sign}(\delta E) \times \frac{\Delta(\delta E)}{\Delta t}$	Yes	Yes	No
Inverse Energy Rate Efficiency	$\frac{V_a(T-D)}{V_c W(\gamma_c + \frac{\dot{V}_c}{g})}$	Yes	Yes	Yes
Maximum Potential Flight Path Angle	$\frac{T_{max} - D}{W}$	No	Yes	No
Minimum Potential Flight Path Angle	$\frac{T_{idle} - D}{W}$	No	Yes	No
Energy Rate Demand (Approach and Landing)	$\frac{W(\gamma_c + \frac{\dot{V}_c}{g})}{T_{idle} - D}$	No	Yes	Yes
Thrust Margin	$1 - \frac{T}{T_{max}}$	No	Yes	No
Energy Rate Margin	$\frac{W(\gamma_a + \frac{\dot{V}_a}{g})}{T_{max} - D}$	No	Yes	No

Summary and Interpretation of Implemented Energy Metrics

Table 1 outlines all the energy metrics implemented in this work along with their formulas. It should be noted that in most cases the metrics are specific metrics, meaning that they have been normalized by the weight. This ensures that they can be used as objective currency to compare a broad range of flights. In this work, the metrics are implemented in the approach and landing phase.

The last column indicates whether a reference profile is required to calculate the value of the metric. Columns 3-5 indicate what data is necessary to evaluate the metric (flight data, or both flight data and performance models). This tabulation assists in choosing energy metrics depending on available resources. The subset of metrics that can be obtained from raw flight data is the most basic kinematic metrics and their rates (which are obtained by numerical differentiation). In addition, if a reference profile is available, then the metrics indicating deviation from this profile can also be evaluated from the raw flight data. The performance models mentioned in earlier sections prove to be essential in evaluating metrics that provide information about limits of the aircraft with respect to energy or energy rates. As such, they are essential for obtaining some energy metrics (rows that contain 'No' in column 3 and 'Yes' in column 4).

In addition to the metrics defined in Table 1, aggregate metrics over the entire approach and landing profile can also be evaluated. For example, for each flight record the average value of the specific total energy during approach and landing can be calculated. These values can be used as a measure of the overall energetic state of the aircraft during approach and landing rather than at a specific point. These metrics are not used in the current work but in a companion paper by the authors.[33]

D. Using Energy Metrics for Detecting Safety Events

Once all the energy metrics have been evaluated for each flight record, they can be visualized and used for safety analysis. In the context of flight data monitoring, a large set of flight data records is assumed to be available. In this work, the flight records from training flights mentioned earlier is utilized. From the point of view of visualization, it is therefore useful to take into account the information contained in the entire data set along with the specific data record being considered. An example was presented earlier in Figure 4 for the specific total energy rate. In the subsequent visualizations, a similar representation will be used for all the metrics and raw parameters. As seen in Figure 5, for each metric during approach, the average is shown with the dashed line, metrics for 50^{th} percentile of flight records are shown with the dark grey band, and metrics for 90^{th} percentile of flight records are shown with the light grey band. Metrics for the particular record under consideration are shown using the solid black line.

For the purpose of demonstrating the use and visualization of energy metrics, two potential unsafe approach and landing scenarios are demonstrated. The first one is a high energy approach and the second one is a low energy approach. In each case, a selected subset of interesting energy metrics is presented to highlight unsafe situations. In a manner similar to the energy metrics, a subset of the raw flight parameters for the same flight record is also presented.

1. ***Case 1: High Energy Approach***

In this subsection, a flight record with high energy during approach and landing is inspected using energy metrics and flight data. Figure 5 shows a visualization of the energy metrics for this flight record. As seen in the figure, the current flight comes in with a specific potential energy and specific kinetic energy much higher than the average. As a result the specific total energy throughout this approach and landing is higher than average and even beyond the 90^{th} percentile. Inspecting the other metrics in Figure 5, it is apparent that even though the specific energy is high, the modified energy error rate is negative for large parts of the approach. This indicates that the energy profile is tending to revert back towards the average. Since the magnitude of this metric is not too high at most places, it indicates that the error is not rapidly increasing or decreasing. This can be seen from the total energy profile which tends to have a slope which is similar to the average. The glide slope for most of the approach is well behaved except towards the end where for a small part, the flight path becomes very steep (upto -10 degrees). This could be an indicator of unsafe operation. The total energy rate and inverse energy rate efficiency indicate what was suspected earlier: the energy rates of this flight record with respect to the reference profile are not ill-behaved. The thrust margin is initially very low but becomes higher towards the end.

A similar visualization of the raw flight parameters for the same flight record can be seen in Figure 6. It can be seen that the pitch angle for the latter part of the descent is far outside the 90^{th} percentile bounds.

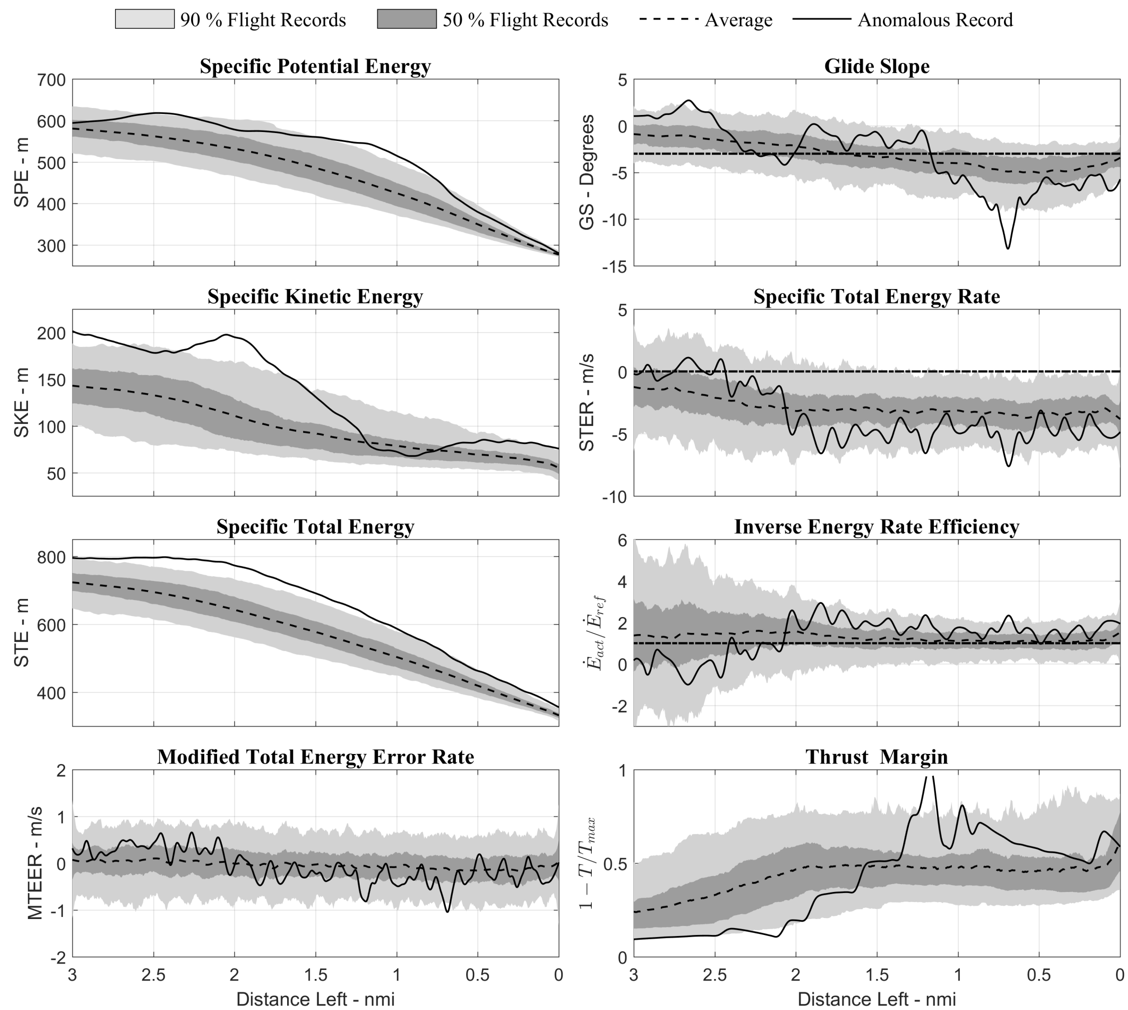


Figure 5. Visualization of energy metrics for a high energy approach

In the initial stages, the RPM and vertical speed are outside the 90^{th} percentile bounds. These could be reasons contributing to the anomalous energy profile seen earlier. The vertical speed towards the end of the approach dips very low (almost -1000 fpm). This is also potentially an unsafe situation. The small blip in RPM between 1.5 and 1 nautical mile left causes the thrust margin in the earlier plot to go to 1, which might point to error in the models or noise in the data.

It is important to note that the behavior of energy metrics was used in this case to single out this flight record. It is a good indicator that flight records with abnormal behavior in energy metrics are those that correspond to specific safety events defined with respect to raw parameters like roll, pitch vertical speed etc.

2. Case 2: Low Energy Approach

In this subsection, a flight record with a very low specific total energy during approach is inspected. As seen from Figure 7, the specific potential and kinetic energies of the flight record are much lower than average.

Unlike the previous record though, the magnitude of the modified energy error rate are quite high between 1.5 and 1 nautical mile distance remaining. This is the region where the energy profile of the current record shifts towards the reference profile (hence the negative value of the metric which is preferred over positive value). In other locations along the approach, the modified energy error rate is much better behaved. Unlike the previous record, the glide slope is not as close to the reference profile or the 3 ° line during approach. At

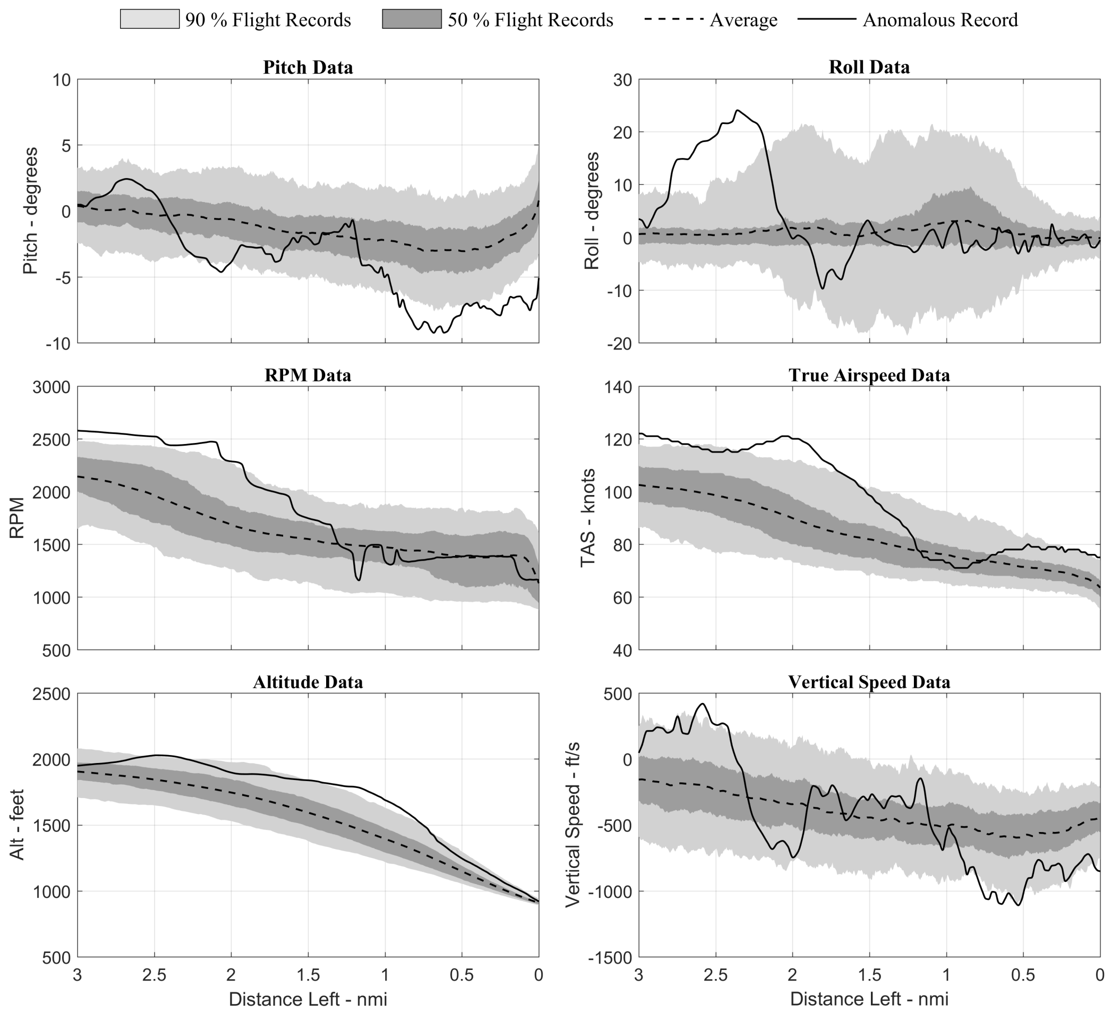


Figure 6. Visualization of raw flight parameters for a high energy approach

some places (where the aircraft is trying to recover from low energy state) the glide slope is even positive. The specific total energy rate and inverse energy rate efficiency indicate a few places where the energy rates are much higher than average and beyond even the 90^{th} percentile values. The thrust margin however is quite uniform throughout the approach without being too low or dangerous in any place. This indicates that even though the aircraft is flying a low energy profile, there is possibly sufficient thrust margin to recover the aircraft from this low energy state (assuming that the engine has not failed).

Similar to the earlier flight record, a visualization of the raw flight parameters is seen in Figure 8. For this particular record the vertical speed and pitch denote abnormal behavior during the approach and landing.

Through the visualization of energy metrics for two cases we have seen how different energy metrics can be used to interpret the energy state of the aircraft from various view points. While a particular record might seem abnormal or anomalous from the point of view of a metric or subset of metrics it may be well behaved for other metrics. This is because different metrics deal with different aspects of the aircraft performance. Therefore, it will be important in future efforts to identify the set or subset of metrics which best capture the aspects of performance that are to be examined.

These two simple examples have shown how energy metrics can be useful in analysis of flight data records. While the examples shown here pertain to approach and landing phase, the same methodology can be extended to other phases of flight. Energy metrics can be a powerful concept in retrospective flight data

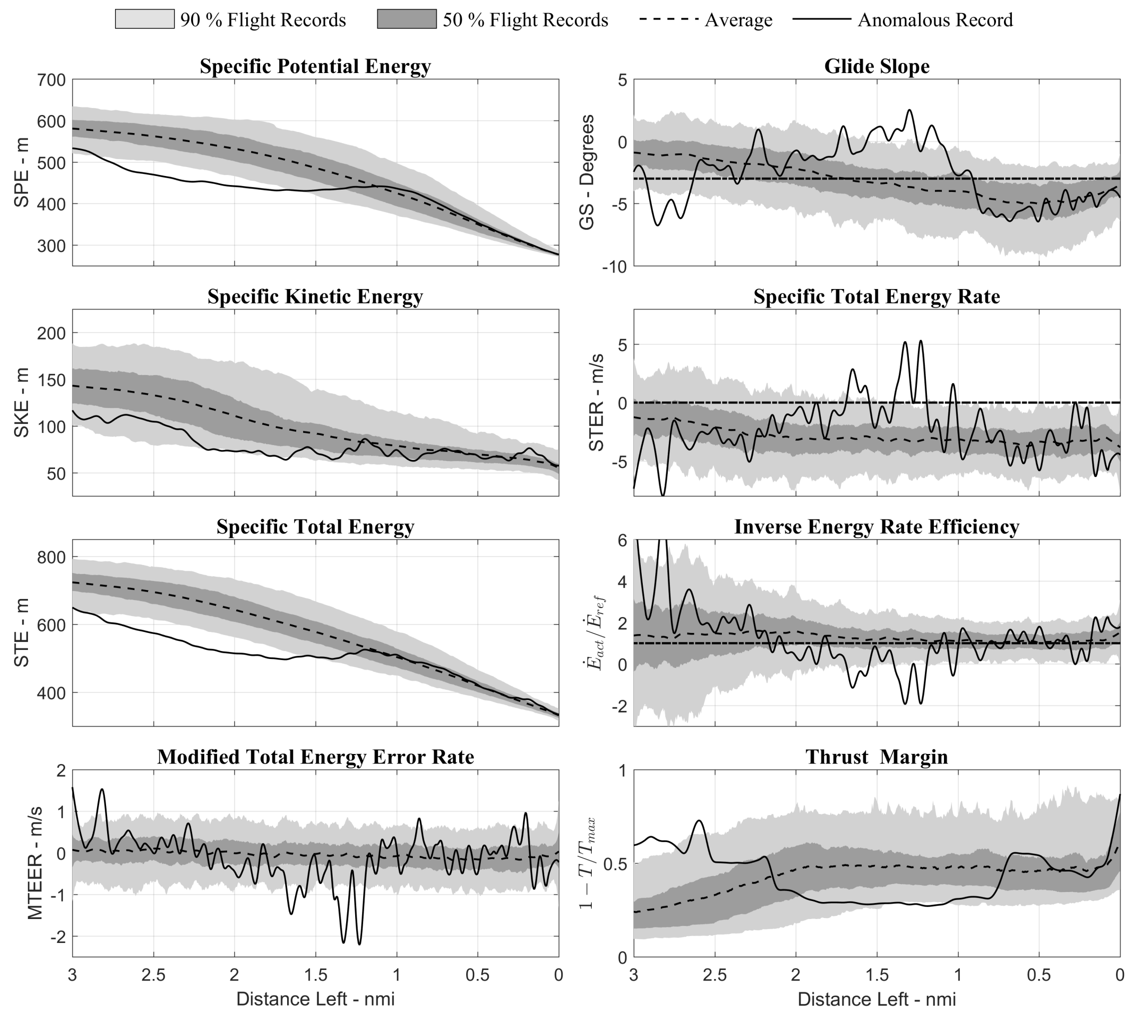


Figure 7. Visualization of energy metrics for a low energy approach

analysis because they not only capture the energetic state of the aircraft but can also be used as an indicator of broader aircraft limits (e.g: Thrust Margin or Energy Rate Margin). The importance of specific energy metrics lies in the fact that these can be generalized more easily as they do not depend on the weight of the aircraft. This means that certain limits of the aircraft operating envelope which vary from one model to another can be captured more broadly by limits on values of specific energy metrics.

V. Conclusions and Future Work

In this paper, we conducted a thorough survey of existing literature on energy management and energy metrics. A few different ways of classifying the existing literature are demonstrated for the readers to understand the existing body of work. Through this classification and further research, we identified important challenges and opportunities for using energy metrics in GA aircraft operations. Modifications to existing metrics and definition of some new energy metrics is proposed. We have demonstrated a methodology for unifying and implementing all the existing metrics and newly defined metrics in a systematic way. Visualization of useful energy metrics along with raw flight data parameters is shown to highlight the potential of using energy metrics in a retrospective flight data analysis setting. Two examples were presented that show the use of energy metrics in a retrospective flight analysis setting.

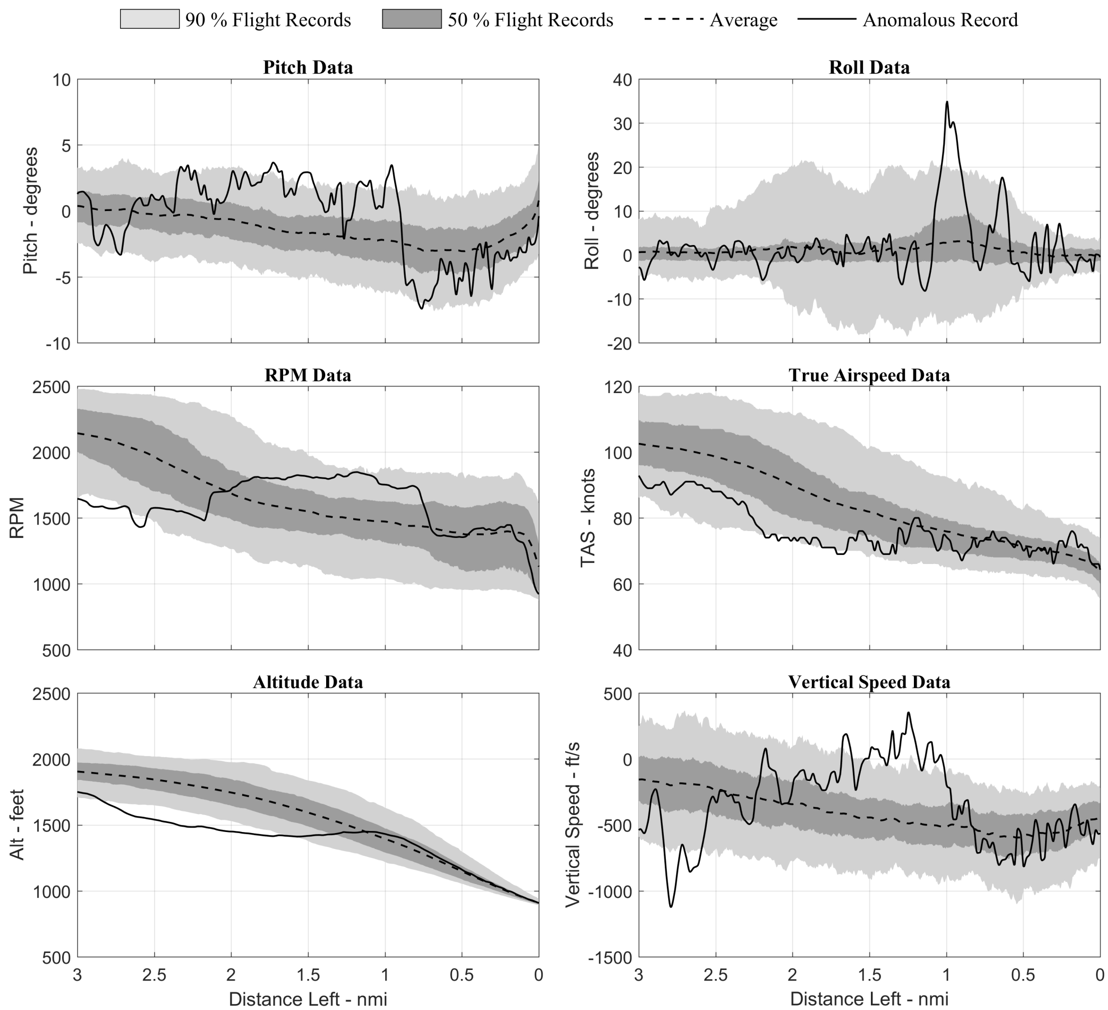


Figure 8. Visualization of raw flight parameters for a low energy approach

In future work, all of these analyses will be extended to other phases of flight to make the methodology more general. With the energy metrics defined and implemented, different ways of utilizing energy metrics for automatically identifying potential 'anomalous' flight data records will be examined. A comparison of the anomalous records identified by energy metrics with traditional exceedance detection methods will be performed to highlight the potential benefits of using energy metrics. Recommendations on which energy metrics prove to be more useful for identifying unsafe events will be provided.

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